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Scattering of thermal waves and non-steady effective thermal conductivity of composites with coated fibers

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Abstract

In this study, thermal wave method is applied to predict the non-steady effective thermal conductivity of composites with coated fibers, and the analytical solution of the problem is obtained. The Fourier heat conduction law is applied to analyze the propagation of thermal waves in the fibrous composite. The scattering and refraction of thermal waves by a cylindrical fiber with coating in the matrix are analyzed, and the results of the single scattering problem are applied to the composite medium. The wave fields in different material layers are expressed by using the wave function expansion method, and the expanded mode coefficients are determined by satisfying the boundary conditions of the layer. The theory of Waterman and Truell is employed to obtain the effective propagating wave number and non-steady effective thermal conductivity of composites. As an example, the effects of the material properties of the coating on the effective thermal conductivity of composites are graphically illustrated and analyzed. Analysis shows that the non-steady effective thermal conductivity under higher frequencies is quite different from the steady thermal conductivity. In the region of lower frequency, the effect of the properties of the coating on the effective thermal conductivity is greater. Comparisons with the steady thermal conductivity obtained from other methods are also presented.

Keywords: Fiber reinforced composite materials; Scattering of thermal waves; Coated fiber; Non-steady effective thermal conductivity

1. Introduction

The subject of the effective thermal conductivity of composites is one of the classical problems in heterogeneous media which has recently draw renewed interest due to the increasing importance of high temperature systems, e.g., car manufacturing, dedicated space structures, etc. These materials usually undergo a complex thermal history. The design of composite materials for such applications requires a thorough understanding of heat conduction in them. The foundation of this understanding lies in the development of micromechanics models for accurately predicting the effective thermal conductivity of multiphase composites [1].

The methods used to measure the thermal conductivity are divided into two groups: the steady state and the non-steady state methods. In the first one, the sample is subjected to a constant heat flow. In the second group, a periodic or transient heat flow is established in the sample [2]. In the past, much attention has been focused on the problems of steady state.

The earliest models for the thermal behavior of composites assumed that the two components are both homogeneous, and are perfectly bounded across a sharp and distinct interface. The Maxwell solution [3] is the starting point to find the effective conductivity of two-phase material systems, but it is valid only for very low concentration of the dispersed phase. Subsequently, many structural models, e.g., Parallel, Maxwell–Eucken [4], and Effective Medium Theory models [5], were proposed. Recently, Samantray et al. applied the unit-cell approach to study the effective thermal conductivity of two-phase materials [6]. The idea of the Generalized Self-Consistent Model was also developed by Hashin [7] to determine the effective thermal conductivity of the two-phase materials.

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Recently, coating inclusions have been introduced in the design of composite to enhance thermal properties. In the modeling, the coating was also introduced for other reasons: first, during the manufacturing process, a chemical reaction between inclusion and matrix can create a third phase: the coating. Second, due to a mismatch between the two phases, the perfect interface assumption is not valid. Thus, the coating contributes to the character of the non-perfect interface. The dramatic effect of interfacial characteristics on thermal conductivities and thermal diffusivities has been experimentally demonstrated by Hasselman et al. in particle [8] and fiber reinforced composites [9]. Based on an equivalent inclusion concept, Hasselman and Johnson extended Maxwell's theory to the systems of spherical and cylindrical inclusions with contact resistance [10]. Benveniste and his co-worker have proposed several analytical models to predict the effective thermal conductivity of composite materials which include the important effects of a thermal contact resistance between the fillers and matrix [11], and the coated cylindrically orthotropic fibers with a prescribed orientation distribution [12]. Lu and Song [13,14] investigated coated or debonded inclusion and developed a more general model to predicting the effective thermal conductivity of composites.

Due to the complexity of non-steady loading, there are few calculations on the effective thermal diffusivity in these materials under modulated conditions. Recently, Monde and Mitsutake [15] proposed a method for determining the thermal diffusivity of solids by using an analytical inverse solution for unsteady heat conduction. By using modulated photothermal techniques, Salazar et al. [2] studied the effective thermal diffusivity of composites made of a matrix filled with aligned circular cylinders of a different material. Most recently, Fang and Hu investigated the distribution of dynamic effective thermal properties along the gradation direction of functionally graded materials by using Fourier heat conduction law [16] and non-Fourier heat conduction law [17].

Nevertheless, no attention has been paid to the non-steady effective thermal conductivity of composites with coated fibers. With the wide application of materials in aerospace and other high temperature situations, the study on the non-steady effective thermal conductivity of composites with coating fibers plays very important role in the designing and manufacture of materials. The main objective of this paper is to investigate the scattering of thermal waves and the effects of coating on the non-steady effective thermal conductivity of materials. Thermal wave is often applied with Fourier conduction law. Fourier' law underlies "parabolic thermal wave" associated with a nonlinear dependence of thermal conductivity on temperature and the "thermal wave method" of measuring thermal properties. The composite medium contains a random distribution of cylindrical inclusions of same size with coating of same thickness. The temperature fields in different regions of the material are expressed by using the wave function expansion method, and the expanded mode coefficients are determined by satisfying the boundary conditions of the coating. The theory of Waterman and Truell [18] is applied to obtain the non-steady effective thermal conductivity of composites. The variation of effective thermal conductivity under different parameters is graphically illustrated and discussed.

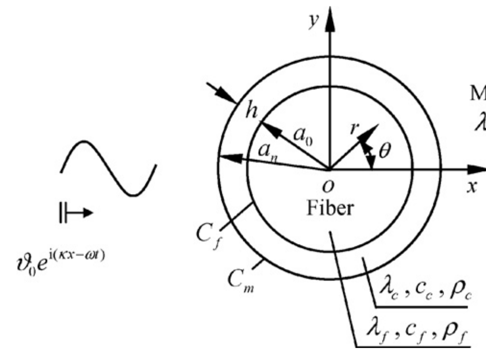

Fig. 1. Coated cylindrical fiber and the incidence of thermal waves in composites.

2. Dynamical equations and their solutions

Consider a composite material containing long, parallel, randomly distributed coated fibers embedded in an infinite matrix [14]. The fibers of radius a_0 have identical properties. Let λ, c, ρ be the thermal conductivity, specific heat capacity and mass density of the matrix, and λ_f, c_f, ρ_f those of the fibers. It is assumed that the thickness of the coating is h with material properties λ_c, c_c, ρ_c.

In order to study the scattering of thermal waves in composite materials with coated fibers, we first consider the scattered field due to a single fiber with coating layer. The geometry is depicted in Fig. 1, where (x, y) is the Cartesian coordinate system with origin at the center of the fiber and (r, θ) is the corresponding cylindrical coordinate system. Let the boundary of the fiber and the coating be denoted by C_f, and that of the coating and the matrix by C_m.

Based on the Fourier heat conduction law, the heat conduction equation in the composite material, in the absence of heat sources, is described as

$$\nabla^2 T(r, t) = \frac{1}{D}\frac{\partial T}{\partial t}, \tag{1}$$

where $\nabla^2/\partial^2 + \partial^2/\partial y^2$ represents the two-dimensional Laplacian operator, T is the temperature in composite materials, D is the thermal diffusivity with

$$D = \frac{\lambda}{\rho c}, \tag{2}$$

where λ, c and ρ are the thermal conductivity, specific heat at constant pressure and density of the matrix, respectively.

The solution of periodic steady state is investigated. Suppose that

$$T = T_0 + \mathrm{Re}[\vartheta \exp(-\mathrm{i}\omega t)], \tag{3}$$

where T_0 is the average temperature, and ω is the incident frequency of thermal waves.

Substituting Eq. (3) into Eq. (1), the following equation can be obtained

$$\nabla^2\vartheta + \kappa^2\vartheta = 0, \tag{4}$$

where κ is the wave number of complex variables in materials, and

$$\kappa = (1+\mathrm{i})k, \tag{5}$$

with $k = \sqrt{\omega/2D}$ being the incident wave number.

By using wave function expansion method, the incident thermal waves are expressed as

$$\vartheta^{(\mathrm{i})} = \vartheta_0\,\mathrm{e}^{\mathrm{i}(\kappa x-\omega t)} = \vartheta_0 \sum_{n=-\infty}^{\infty} \mathrm{i}^n J_n(\kappa r)\,\mathrm{e}^{\mathrm{i}n\theta}\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{6}$$

where the superscript (i) stands for the incident waves in the matrix, ϑ_0 is the temperature amplitude of incident thermal waves in the matrix, and $J_n(\cdot)$ are the nth Bessel functions of the first kind. It should be noted that all wave fields have the same time variation $\mathrm{e}^{-\mathrm{i}\omega t}$, which is omitted in all subsequent representations for notational convenience.

When the thermal waves propagate in the fibrous composite material, the waves are scattered by the fibers, and the scattered waves of the fibers are expanded in a series of outgoing Hankel functions. The scattered field in the matrix is expressed in the forms

$$\vartheta^{(\mathrm{s})} = \sum_{n=-\infty}^{\infty} A_n H_n^{(1)}(\kappa r)\,\mathrm{e}^{\mathrm{i}n\theta}, \tag{7}$$

where the superscript (s) stands for the scattered waves, $H_n^{(1)}(\cdot)$ are the nth Hankel functions of the first kind, and A_n are the mode coefficients that account for the distortion of the scattered cylindrical waves by the fiber.

The total temperature in the matrix should be produced by the superposition of the incident field and the scattered field, i.e.,

$$\vartheta^{\mathrm{m}} = \vartheta^{(\mathrm{i})} + \vartheta^{(\mathrm{s})}. \tag{8}$$

The refracted waves inside the fiber are standing waves, and can be expressed as

$$\vartheta^{\mathrm{r}} = \sum_{n=-\infty}^{\infty} B_n J_n(\kappa_{\mathrm{f}} r)\,\mathrm{e}^{\mathrm{i}n\theta}, \tag{9}$$

where the superscript r stands for the refracted waves, and B_n are the mode coefficients of refracted waves.

The temperature in the coating ϑ^c may be described by the sum of the two components (outgoing and ingoing) and is expressed in the following form [19,20]

$$\vartheta^{\mathrm{c}} = \left[\sum_{n=-\infty}^{\infty} E_n H_n^{(1)}(\kappa_{\mathrm{c}} r)\,\mathrm{e}^{\mathrm{i}n\theta} + \sum_{n=-\infty}^{\infty} F_n H_n^{(2)}(\kappa_{\mathrm{c}} r)\,\mathrm{e}^{\mathrm{i}n\theta}\right], \tag{10}$$

where $H_n^{(2)}$ are the nth Hankel functions of the second kind, and denote the ingoing waves, and E_n and F_n are the mode coefficients in the coating.

The wave numbers κ_{c} in the coating and κ_{f} in the cylindrical fiber are given by

$$\kappa_{\mathrm{c}} = (1+\mathrm{i})\sqrt{\frac{\omega}{2D_{\mathrm{c}}}}, \tag{11}$$

$$\kappa_{\mathrm{f}} = (1+\mathrm{i})\sqrt{\frac{\omega}{2D_{\mathrm{f}}}}, \tag{12}$$

where $D_{\mathrm{c}} = \lambda_{\mathrm{c}}/\rho_{\mathrm{c}}c_{\mathrm{c}}$ and $D_{\mathrm{f}} = \lambda_{\mathrm{f}}/\rho_{\mathrm{f}}c_{\mathrm{f}}$.

3. Boundary conditions and solution of the coefficients

The boundary conditions on C_m and C_{f} are given by

$$\vartheta^{\mathrm{c}} = \vartheta^{\mathrm{m}}, \qquad q_{\mathrm{r}}^{\mathrm{c}} = q_{\mathrm{r}}^{\mathrm{m}} \quad \text{for} \quad r = a_n, \tag{13}$$

$$\vartheta^{\mathrm{r}} = \vartheta^{\mathrm{c}}, \qquad q_{\mathrm{r}}^{\mathrm{r}} = q_{\mathrm{r}}^{\mathrm{c}} \quad \text{for} \quad r = a_0, \tag{14}$$

where q_{r} is the heat flow density in the radial direction, and $q_{\mathrm{r}} = -\lambda(\partial\vartheta/\partial r)$.

The continuous boundary condition of temperature on C_m gives

$$\sum_{n=-\infty}^{\infty} [E_n H_n^{(1)}(\kappa_{\mathrm{c}} a_n)\,\mathrm{e}^{\mathrm{i}n\theta} + F_n H_n^{(2)}(\kappa_{\mathrm{c}} a_n)\,\mathrm{e}^{\mathrm{i}n\theta}$$
$$= \vartheta_0 \sum_{n=-\infty}^{\infty} \mathrm{i}^n J_n(\kappa a_n)\,\mathrm{e}^{\mathrm{i}n\theta} + \sum_{n=-\infty}^{\infty} A_n H_n^{(1)}(\kappa a_n)\,\mathrm{e}^{\mathrm{i}n\theta}, \tag{15}$$

Multiplying by $\mathrm{e}^{-\mathrm{i}s\theta}$ and integrating from 0 to 2π, the following equation can be obtained

$$E_s H_s^{(1)}(\kappa_{\mathrm{c}} a_n) + F_s H_s^{(2)}(\kappa_{\mathrm{c}} a_n) = \vartheta_0 i^{\mathrm{s}} J_s(\kappa a_n) + A_s H_s^{(1)}(\kappa a_n), \tag{16}$$

The continuous boundary conditions of temperature on C_{f} give

$$B_s J_s(\kappa_{\mathrm{f}} a_0) = E_s H_s^{(1)}(\kappa_{\mathrm{c}} a_0) + F_s H_s^{(2)}(\kappa_{\mathrm{c}} a_0). \tag{17}$$

According to the continuous boundary conditions of heat flux density on C_m and C_{f}, one can obtain

$$\lambda_{\mathrm{c}}\left[E_s \frac{\partial}{\partial a_n} H_s^{(1)}(\kappa_{\mathrm{c}} a_n) + F_s \frac{\partial}{\partial a_n} H_s^{(2)}(\kappa_{\mathrm{c}} a_n)\right]$$
$$= \lambda\left[A_s \frac{\partial}{\partial a_n} H_s^{(1)}(\kappa a_n) + \vartheta_0 i^{\mathrm{s}} \frac{\partial}{\partial a_n} J_s(\kappa a_n)\right], \tag{18}$$

$$\lambda_{\mathrm{f}}\left[B_s \frac{\partial}{\partial a_0} J_s(\kappa_{\mathrm{f}} a_0)\right] = \lambda_{\mathrm{c}}\left[E_s \frac{\partial}{\partial a_0} H_s^{(1)}(\kappa_{\mathrm{c}} a_0)\right.$$
$$\left. + F_s \frac{\partial}{\partial a_0} H_s^{(2)}(\kappa_{\mathrm{c}} a_0)\right], \tag{19}$$

After some manipulations, Eqs. (16)–(19) can be arranged as

$$[P]\{X\} = \{Q\}, \tag{20}$$

where $X = A_s$, B_s, E_s, F_s, P is a coefficient matrix of 4×4, and f is a vector of 4 ranks, whose elements are shown in Appendix

A. After solving the linear equation system (20), the mode coefficients A_s, B_s, E_s, F_s ($s = 0, \pm1, \pm2, \ldots$) ($s = 0, 1, 2, \ldots$) can be obtained.

4. Effective propagating wave number of thermal waves

We now consider a composite material with N fibers randomly distributed in the matrix. Their positions of these fibers are denoted by the random variables (r_1, r_2, ..., r_N). The total temperature field at any point outside all fibers can be given in the multiple scattering form

$$\vartheta(r; r_1, r_2, \ldots r_N) = \vartheta^{\mathrm{i}}(r) + \sum_{k=1}^{N} T^{\mathrm{s}}\vartheta^{\mathrm{i}}(r_k) + \sum_{m=1}^{N} T^{\mathrm{s}}(r_m) \sum_{k=1, k\neq m}^{N} T^{\mathrm{s}}(r_k)\vartheta^{\mathrm{i}}(r_m) + \cdots \tag{21}$$

where the single summation denotes the primary scattered terms, the double summation denotes the secondary terms and so on. The primary scattering is due to the incident waves alone, and the second scattering represents the rescattering of the primary scattered waves, etc. The multiple scattering theory takes into account the interaction among the distributed fibers accurately. However, it is difficulty to deal with in order to predict the effective properties. Here, we apply the effective field approximation to describe approximately the interaction among the distributed fibers. Following the work of Waterman and Truell [18], the effective propagating wave number can be obtained from the scattered far field.

Once the scattered field due to a single fiber is known, the phase velocities and attenuations of the coherent waves through the composite can be easily calculated by the double plane wave theory of Waterman and Truell [18]. The scattered fields for incident thermal waves at a large distance from the particle can be obtained from Eq. (7) by letting r tend to ∞. After applying the asymptotic expression of the radial function $H_n^{(1)}(\kappa r)$, the scattered wave in the far-fields can be expressed asymptotically

$$\vartheta_{\mathrm{r}}^{(\mathrm{s})} \sim \sqrt{\frac{2}{\pi\kappa r}}\, \mathrm{e}^{\mathrm{i}(\kappa r - \pi/4)}(\mathrm{i}\kappa\vartheta_0) f(\kappa, \theta) + o\left(\frac{1}{r}\right), \tag{22}$$

where

$$f(\kappa, \theta) = \sum_{s=-\infty}^{\infty} (-\mathrm{i})^{\mathrm{s}} \frac{A_s}{\vartheta_0}\, \mathrm{e}^{\mathrm{i}s\theta}. \tag{23}$$

The function $f(\kappa, \theta)$ is the far-field scattering amplitudes for the scattered thermal waves. It is noted that the far-field scattered amplitudes are dependent on the angle θ. The far-field scattered amplitudes at two specific angles, $\theta = 0$ and $\theta = \pi$, are of special interest, and are called the forward and backward scattering amplitudes, respectively.

According to the theory of Waterman and Truell [18], in the case of two-dimensional scatterers, the effective propagating wave number is expressed as

$$\left(\frac{K}{k}\right)^2 = \left[1 - \frac{2\mathrm{i}N}{k^2} f(\kappa, 0)\right]^2 - \left[\frac{2\mathrm{i}N}{k^2} f(\kappa, \pi)\right]^2, \tag{24}$$

where K is the propagating wave number in the effective medium, and N is the number of the fibers per unit volume with

$$N = \frac{V_{\mathrm{f}}}{\pi a_0^2}, \tag{25}$$

in which V_{f} is the volume fraction of the randomly distributed cylindrical fibers in the matrix.

It is noted that $f(\kappa, 0)$ is the forward scattering amplitude of a single scatterer, and $f(\kappa, \pi)$ is the backward scattering amplitude of a single scatterer. In the theory of Waterman and Truell [20], correlations between the fibers are neglected. Thus, the validity of Eq. (24) is limited to the low volume concentration of fibers. In the region of higher frequency, the value of V_{f} should be $V_{\mathrm{f}} \leq 0.2$. With the decrease of wave frequency, a greater value of V_{f} can be chosen. However, it should not be greater than 0.3.

5. Non-steady effective properties of the fiber-reinforced composites

According to Eq. (5), the non-steady effective thermal conductivity λ^{eff} can be easily obtained from the effective propagating wave number as follows:

$$\lambda^{\mathrm{eff}} = \frac{\rho^{\mathrm{eff}} c^{\mathrm{eff}} \lambda}{\rho c} \left[\mathrm{Re}\left(\frac{k}{K}\right)\right]^2 \tag{26}$$

where $\mathrm{Re}(\cdot)$ denotes the real part, and ρ^{eff} and c^{eff} are the effective mass density and effective heat capacity of composites. From Ref. [2], it is known that ρ^{eff} and c^{eff} always follow the mixture rule, and $\rho^{\mathrm{eff}} c^{\mathrm{eff}}$ is given by

$$\rho^{\mathrm{eff}} c^{\mathrm{eff}} = \rho c \left\{1 - V_{\mathrm{f}}\left(1 + \frac{h}{a_0}\right)^2\right\} + \rho_{\mathrm{f}} c_{\mathrm{f}} V_{\mathrm{f}} + \frac{h V_{\mathrm{f}}}{a_0} \rho_{\mathrm{c}} c_{\mathrm{c}} \left(2 + \frac{h}{a_0}\right). \tag{27}$$

6. Numerical examples and discussion

In the following analysis, it is convenient to make the variables dimensionless. To accomplish this step, a representative length scale a_0, where a_0 is the radius of fibers, is introduced. The following dimensionless variables and quantities have been chosen for computation: the incident wave number $k^* = ka_0 = 0.1$–2.0, $h^* = ha_0 = 0.05$–0.20, $\lambda_{\mathrm{f}}^* = \lambda_{\mathrm{f}}/\lambda = 2.0$–$8.0$, $c_{\mathrm{f}}^* = c_{\mathrm{f}}/c = 2.0$–$4.0$, $\rho_{\mathrm{f}}^* = \rho_{\mathrm{f}}/\rho = 2.0$–$4.0$, $\lambda_{\mathrm{c}}^* = \lambda_{\mathrm{c}}/\lambda = 0.5$–$8.0$, $c_{\mathrm{c}}^* = c_{\mathrm{c}}/c = 1.0$–$4.0$, and $\rho_{\mathrm{c}}^* = \rho_{\mathrm{c}}/\rho = 1.0$–$4.0$. The dimensionless effective thermal conductivity is $\lambda^* = \lambda^{\mathrm{eff}}/\lambda$. During computation, it is found that it is numerically sufficient to truncate s at 8 for any desired incident frequency.

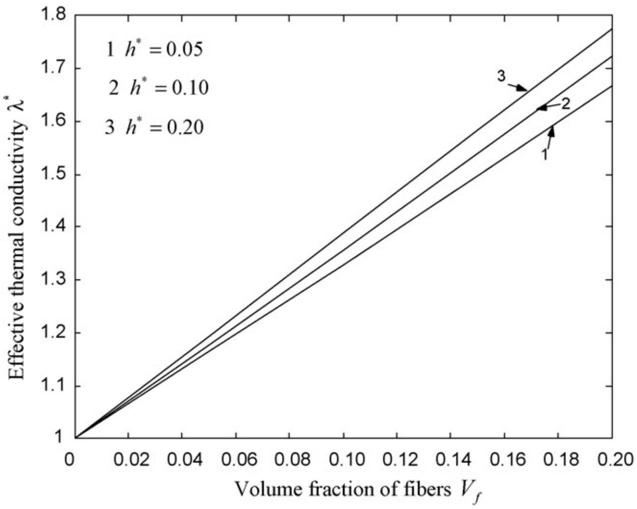

Fig. 2. Non-steady effective thermal conductivity as a function of volume fraction of fibers ($k^* = 1.0$, $\lambda_f^* = 4.0$, $c_f^* = \rho_f^* = 2.0$, $\lambda_c^* = 2.5$, $c_c^* = \rho_c^* = 1.5$).

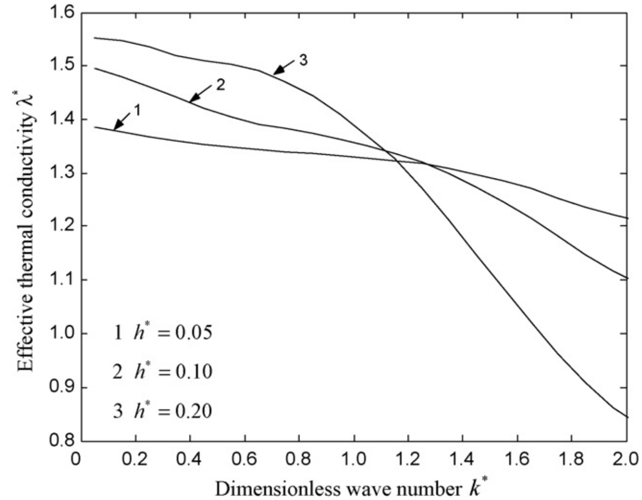

Fig. 4. Non-steady effective thermal conductivity as a function of dimensionless wave number ($V_f = 0.1$, $\lambda_f^* = 4.0$, $c_f^* = \rho_f^* = 2.0$, $\lambda_c^* = 2.5$, $c_c^* = \rho_c^* = 1.5$)..

The non-steady effective thermal conductivity of composites as a function of volume fraction of fibers with parameters: $k^* = 1.0$, $\lambda_f^* = 4.0$, $c_f^* = \rho_f^* = 2.0$, $\lambda_c^* = 2.5$, $c_c^* = \rho_c^* = 1.5$ is presented in Fig. 2. It can be seen that the non-steady effective thermal conductivity increases with the increase of the coating thickness. Because the thermal conductivity of the fiber is greater than that of the matrix, the non-steady effective thermal conductivity increases with the volume fraction of fibers. The effect of the coating thickness on the effective thermal conductivity also increases with the volume fraction of fibers. From Eqs. (26) and (27), it is found that the non-steady effective thermal conductivity increases with the increase of the values of c_f^* and ρ_f^*. The effects of the values of c_f^* and ρ_f^* on the effective thermal conductivity also increase with the volume fraction of fibers. It is known that the steady effective thermal conductivity is not dependent on the specific heat and density of the two phases. However, the specific heat and density of the two phases have great effect on the dynamic effective thermal conductivity of composites.

Fig. 3 illustrates the non-steady effective thermal conductivity of composites as a function of the incident wave number with parameters: $h^* = 0.1$, $V_f = 0.1$, $\lambda_f^* = 4.0$, $\lambda_c^* = 2.5$, $c_c^* = \rho_c^* = 1.5$. It can be seen that in the region of low frequency, the variation the specific heat and density of the two phases nearly expresses no effect on the effective thermal conductivity. With the increase of the incident wave number, the effect of the specific heat and density of the two phases on the non-steady effective thermal conductivity increases greatly. The non-steady effective thermal conductivity increases with the specific heat and density ratio of the fiber and matrix.

Fig. 4 illustrates the non-steady effective thermal conductivity of composites as a function of dimensionless wave

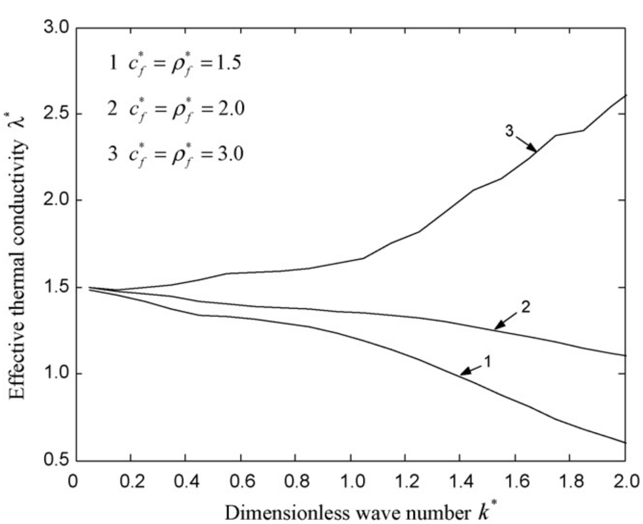

Fig. 3. Non-steady effective thermal conductivity as a function of dimensionless wave number ($h^* = 0.1$, $V_f = 0.1$, $\lambda_f^* = 4.0$, $\lambda_c^* = 2.5$, $c_c^* = \rho_c^* = 1.5$)..

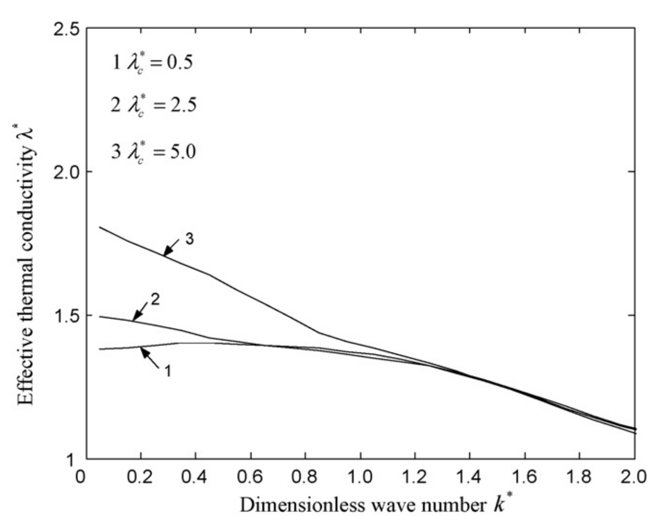

Fig. 5. Non-steady effective thermal conductivity as a function of dimensionless wave number ($V_f = 0.1$, $h^* = 0.1$, $\lambda_f^* = 4.0$, $c_f^* = \rho_f^* = 2.0$, $c_c^* = \rho_c^* = 1.5$).

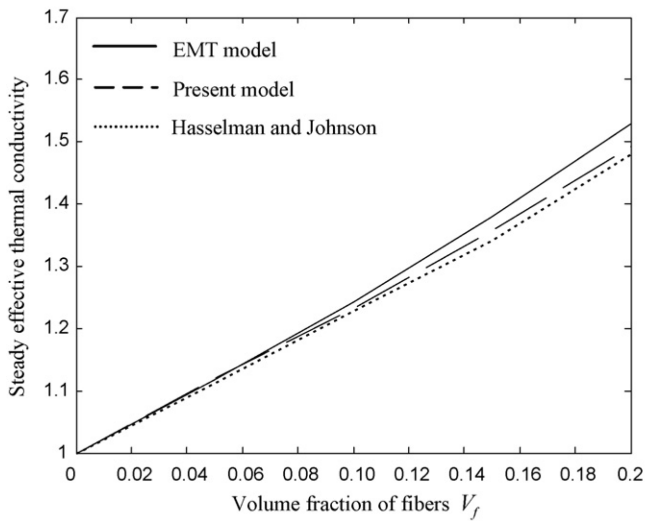

Fig. 6. Comparison of the steady effective thermal conductivity with EMT model and Hasselman and Johnson (Ref. [10]) ($\lambda_f^* = 4.0$, $c_f^* = 2.0$, $\rho_f^* = 2.0$, $h^* = 0$, $k^* = 0$).

number with parameters: $V_f = 0.1$, $\lambda_f^* = 4.0$, $c_f^* = \rho_f^* = 2.0$, $\lambda_c^* = 2.5$, $c_c^* = \rho_c^* = 1.5$. It can be seen that in the region of low frequency the non-steady effective thermal conductivity increases with the increase of the value of h^*. However, in the region of high frequency, the non-steady effective thermal conductivity decreases with the increase of the value of h^*.

Fig. 5 shows the non-steady effective thermal conductivity of composites as a function of dimensionless wave number with parameters: $V_f = 0.1$, $h^* = 0.1$, $\lambda_f^* = 4.0$, $c_f^* = \rho_f^* = 2.0$, $c_c^* = \rho_c^* = 1.5$. It can be seen that in the region of low frequency the non-steady effective thermal conductivity increases with the increase of the value of λ_c^*. However, in the region of high frequency, the non-steady effective thermal conductivity nearly expresses no variation with the value of λ_c^*.

Finally, to demonstrate the validity of this dynamic thermal model, the steady effective thermal conductivity of two-phase composites without coating is given. As $ka_0 \to 0$, the dynamic effective thermal conductivity tends to the steady solutions. In Fig. 6, the results obtained from the present model, Effective Medium Theory [5] and Hasselman and Johnson [10] are plotted. It is noted that the steady effective thermal conductivity equations obtained from Effective Medium Theory [5] and Hasselman and Johnson [10] are listed in Appendix B. Close agreement is seen to exist between the models at low volume fractions; however, the present model predicts a lower value of effective thermal conductivity than the Effective Medium Theory. This is consistent with regards to criticism of the conventional Effective Medium Theory for overestimating the effective thermal conductivity of two-phase composites when $\lambda_f > \lambda$. This is attributed to the assumption that the fibers are regarded as the effective medium even at close range.

7. Conclusions

The scattering of thermal waves in composites with coated fibers is investigated theoretically by employing wave functions expansion method. The analytical solution of the non-steady effective thermal conductivity of the composite is presented. The theory of Waterman and Truell is applied to obtain the effective propagating wave number of thermal waves. Comparison with the steady effective thermal conductivity demonstrates the validity of the dynamical thermal model.

It has been found that the non-steady effective thermal conductivity of the composites is dependent on the incident wave number, the material properties ratio of the fiber and matrix and the properties of the coating. The non-steady effective thermal conductivity of the composites increases with an increase of the thickness of the coating, and the thermal conductivity ratio of the fiber and matrix. In contrast to the steady case, the frequency of the thermal waves has great influence on the effective thermal conductivity. In the region of low frequency, the variation the specific heat and density of the two phases nearly expresses no effect on the effective thermal conductivity. With the increase of the incident wave number, the effects of the specific heat and density of the two phases on the non-steady effective thermal conductivity increase greatly. In different region of frequency, the effect of the thickness of the coating also shows great difference. Therefore, to gain a higher effective thermal conductivity of composites, when the frequency of thermal loading is low, the greater thickness and thermal conductivity of the coating and the greater thermal conductivity ratio of the fibers and matrix should be chosen. However, in the region of high frequency ($k^* > 1.0$), the smaller thickness of the coating is preferable.

The results of this paper can provide guidelines for the design of fiber reinforced composites in the presence of coating and would be helpful in understanding the thermal behavior of composites.

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Appendix A

The expressions of P and Q are given by

$$P(1,1) = -H_s^{(1)}(\kappa a_n), \tag{A.1}$$

$$P(1,2) = 0, \tag{A.2}$$

$$P(1,3) = H_s^{(1)}(\kappa_c a_n), \tag{A.3}$$

$$P(1,4) = H_s^{(2)}(\kappa_c a_n), \tag{A.4}$$

$$P(2,1) = 0, \tag{A.5}$$

$$P(2,2) = -J_s(\kappa_0 a_0), \tag{A.6}$$

$$P(2,3) = H_s^{(1)}(\kappa_c a_0), \tag{A.7}$$

$$P(2,4) = H_s^{(2)}(\kappa_c a_0), \tag{A.8}$$

$$P(3,1) = \lambda[sH_s^{(1)}(\kappa a_n) - \kappa a_n H_{s+1}^{(1)}(\kappa a_n)], \tag{A.9}$$

$$P(3,2) = 0, \tag{A.10}$$

$$P(3,3) = \lambda_c[sH_s^{(1)}(\kappa_c a_n) - \kappa_c a_n H_{s+1}^{(1)}(\kappa_c a_n)], \tag{A.11}$$

$$P(3,4) = \lambda_c[sH_s^{(2)}(\kappa_c a_n) - \kappa_c a_n H_{s+1}^{(2)}(\kappa_c a_n)], \tag{A.12}$$

$$P(4,1) = 0, \tag{A.13}$$

$$P(4,2) = -\lambda_0[sJ_s(\kappa_0 a_0) - \kappa_0 a_0 J_{s+1}(\kappa_0 a_0)], \tag{A.14}$$

$$P(4,3) = \lambda_c[sH_s^{(1)}(\kappa_c a_0) - \kappa_c a_0 H_{s+1}^{(1)}(\kappa_c a_0)], \tag{A.15}$$

$$P(4,4) = \lambda_c[sH_s^{(2)}(\kappa_c a_0) - \kappa_c a_0 H_{s+1}^{(2)}(\kappa_c a_0)], \tag{A.16}$$

$$Q(1) = \vartheta_0 \mathrm{i}^s J_s(\kappa a_n), \tag{A.17}$$

$$Q(2) = 0, \tag{A.18}$$

$$Q(3) = \vartheta_0 \mathrm{i}^s[sJ_s(\kappa a_n) - \kappa a_n J_{s+1}(\kappa a_n)], \tag{A.19}$$

$$Q(4) = 0, \tag{A.20}$$

Appendix B

The effective thermal conductivity equation obtained from Effective Medium Method model [5] is expressed as

$$V_f \frac{\lambda_f - \lambda^{eff}}{\lambda_f + 2\lambda^{eff}} + (1 - V_f)\frac{\lambda - \lambda^{eff}}{\lambda + 2\lambda^{eff}} = 0. \tag{B.1}$$

The effective thermal conductivity equation obtained from Hasselman and Johnson [10] is expressed as

$$\lambda^{eff} = \lambda \frac{(\lambda_f/\lambda - 1)V_f + (1 + \lambda_f/\lambda)}{(1 + \lambda_f/\lambda)V_f + (1 + \lambda_f/\lambda)}. \tag{B.2}$$

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